EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS
Earthquake Engng Struct. Dyn. 2000; **29**:327–337

Empirical formula for fundamental vibration periods of reinforced concrete buildings in Taiwan

Li-Ling Hong*,† and Woei-Luen Hwang‡

Department of Civil Engineering, National Cheng Kung University, Tainan 701, Taiwan

SUMMARY

More than 30 buildings around Taiwan have been selected to monitor the floor responses under seismic excitation. The structural array monitoring system in each building controls at most 27 channels of accelerometers distributed in several floors. Those buildings were triggered by many events during the past five years of operation. In each building, the records at the basement can be considered as the ground excitation, and the others at the upper floors are the structural responses. The frequency transfer functions of those buildings can be identified by ARX models, and then the fundamental vibration periods are estimated. The identified fundamental vibration periods using different events are compared in order to ensure the reliability of system identification. An empirical formula in predicting the fundamental vibration period is presented through the regression analysis to the identified fundamental vibration periods of 21 reinforced concrete (RC) moment-resisting frame (MRF) buildings. It is found that the height of a building plays an important role in predicting the fundamental vibration period, compared with the length, width, and time after completion of the building. It is also found that the RC MRF buildings in Taiwan tend to be stiffer than those in the U.S. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS: fundamental vibration period; reinforced concrete moment-resisting frame; system identification; ARX model; frequency transfer function; regression analysis; Uniform Building Code

INTRODUCTION

In most static design methods, the equivalent seismic lateral force is determined from the design spectrum [1], and hence is a function of the fundamental vibration period of the building. There

* Correspondence to: Li-Long Hong, Department of Civil Engineering, National Cheng Kung University, Tainan 701, Taiwan.
† Associate Professor.
‡ Former Graduate Student.

Contract/grant sponsor: Central Weather Bureau, Ministry of Transportation and Communications, R.O.C.; contract/grant number: CWB 86-2E-11.
Contract/grant sponsor: National Science Council, R.O.C.; contract/grant number: NSC 86-2621-p-006-023.

Received 1 September 1998
Revised 25 May 1999
Accepted 5 September 1999

are two ways to determine the fundamental vibration period of a structure. The theoretical approach needs a simplified model in constituting the mass and stiffness matrices and also in evaluating the elements of these matrices. The practical approach uses some measurements in the real structure to identify its fundamental vibration period. For design purpose, the UBC-97 also provides some empirical formulas to evaluate the fundamental vibration periods of buildings of different types. In a period of five years, many structural array monitoring systems were installed in the buildings of different heights around Taiwan island, and the recorded accelerograms offer the database to identify the fundamental vibration periods. Such identified periods can be used to validate the empirical formula supplied by the code or constitute a new empirical formula if necessary.

Jacobsen [2] used a cantilever beam of box section, constant stiffness, and uniform mass distribution to model a building and decided the frequency ratios of the higher modes to the fundamental mode in the two special cases of pure shear and pure bending. Salvadori and Heer [3] assumed the linear variation of stiffness in a cantilever beam and put a lumped mass at its free end to simulate the penthouse of a building. They obtained an empirical formula of the fundamental vibration period, which is proportional to the product of the building height and the reciprocal of the square root of the building width parallel to the direction of vibration. Housner and Brady [4] used the Rayleigh's method to derive many formulas in predicting the fundamental vibration periods of buildings with different distributions of mass and stiffness. New lightweight material and design philosophy, for example, the concept of strong column and weak beam, have been developed for tall buildings. Hence, the assumption of infinite-stiffness beams in the model of shear buildings is no longer accepted in modern buildings. Suko and Adams [5] then introduced the beam-column stiffness ratio to modify the empirical formula. For comparing with the code formulas, Goel and Chopra [6] collected the measured periods of 27 RC MRF buildings and 42 steel MRF buildings in California. They also recommended new empirical formulas in predicting the fundamental vibration periods of RC and steel MRF buildings by performing the regression analysis to the measured periods.

There are many methods in system identification [7]. When compared with the other parameters, the fundamental vibration period is the easiest and also the most acceptable one to be identified, and the identified values given by different methods are most consistent. Therefore, the choice of methods is less important than that of data types used in the identification of the fundamental vibration period. For the purpose of finding the modal parameters of buildings, the seismic records are generally better than those excited by the ambient disturbance and the vibration generator if the vibration is still in the linearly elastic range under ground excitation. The objective of this paper is to identify the fundamental vibration periods of the RC MRF buildings in Taiwan using the **a**uto**r**egressive e**x**ogeneous (ARX) model, then to compare them with the code formula in UBC-97, and finally to suggest a new empirical formula.

STRUCTURAL ARRAY MONITORING SYSTEM USED IN TAIWAN

The structural array monitoring systems operated by the Central Weather Bureau, Ministry of Transportation and Communications, R.O.C. have been installed in several tens of buildings around Taiwan. Each system contains a central recording system supplied by two personal computers for real-time processing and others. Each central recording system accommodates at

Table I. Summary of buildings in system identification.

Building No.	Height (m)	Length (m)	Width (m)	Type	Year of completion	T_1 (sec)	T_t (sec)
1	8.45	71	10.98	RC	1993	0.186	0.146
2	13.56	55.52	52.03	RC	1983	0.248	0.341
3	14.2	97	10	RC	1986	0.423	0.293
4	20	57.6	21.6	RC	1992	0.271	0.299
5	22.95	69.15	27.3	RC	1984	0.375	0.303
6	24.36	56.7	22.82	RC	1983	0.515	0.316
7	24.5	81.6	57.5	RC	1986	0.265	0.239
8	26.8	52.5	37.5	RC	1992	0.325	0.327
9	27.4	53.35	25.95	RC	1992	—	0.540
10	28.5	79.2	14.5	RC	1970	0.360	0.511
11	29.15	64.34	16.5	RC	1979	0.490	0.796
12	31.5	52.72	15	RC	1993	0.397	0.409
13	32.96	64	43.2	RC	1989	0.414	0.478
14	37.89	59.55	35.4	RC	1994	0.544	0.491
15	38.4	48.1	30.4	SRC	1990	0.507	0.521
16	42	97.4	56.28	RC	1992	0.462	0.503
17	42.6	46.5	28.02	RC	1991	0.614	0.500
18	59.5	28.65	21.3	RC	1992	0.662	0.804
19	59.5	30	20.06	RC	1992	0.659	0.778
20	69.55	89.35	36	SRC	1994	1.427	1.373
21	77.1	51.64	17	RC	1995	0.877	1.449

— Unable to identify.

most 30 channels of accelerometers, which could be mounted on the floors and at the free field. A typical accelerograph includes three channels at the free field and the others at the first basement, ground level, roof level, and other middle floors in a building. In each measured floor, there are one channel in the longitudinal direction and two or three channels in the transverse direction.

The installation of the array began in 1992 and was completed in 1996. Twenty-one RC and SRC MRF buildings were triggered by more than four events since operation. The dimensions of those buildings are listed in Table I, and the distribution of heights is shown in Figure 1. Those buildings are beam-column MRF systems except a few shear walls surrounding the elevators. Those buildings were originally selected to install the arrays on purpose to cover different heights and site conditions. Therefore, the data set of identified periods is quite valuable as the almost linear variation of the height is shown in Figure 1.

For each building, the records of four events are selected to identify the fundamental vibration period. In additional to the ground vibration level, the wider separation of the occurring time is also the major concern in selecting these events. The occurring times of these events are from 1993 to 1996. The peak values of the acceleration records at the ground and the floors range from several gals to a few hundred gals. Obviously, these events are not strong shaking. However, they could just be in the scale to ensure the linearly elastic responses, which are important in identifying the fundamental vibration periods.

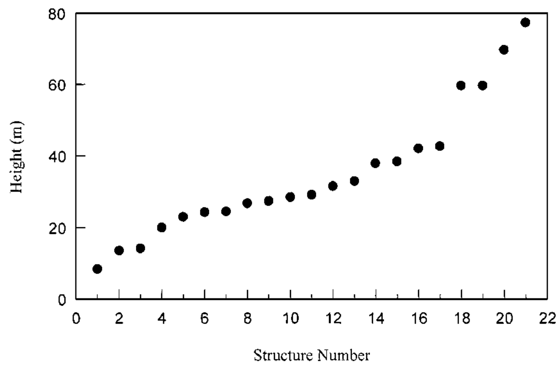

Figure 1. Distribution of building heights.

AUTOREGRESSIVE EXOGENEOUS MODEL

The ARX model is one of the transfer-function models connecting the input and output data in the discrete time domain [8]. For a linear time-invariant system, the ARX model is given as follows:

$$y(t) + a_1 y(t - \Delta t) + \cdots + a_m y(t - m\Delta t) = b_1 u(t - \Delta t) + \cdots + b_n u(t - n\Delta t) + e(t) \tag{1}$$

in which Δt is the time step of the discrete-time signals, m is the number of poles, $n - 1$ is the number of zeros, and a_i and b_i are the system parameters. In Equation (1), the current response, $y(t)$, is related to a finite number of previous excitations, $u(t - \Delta t)$ to $u(t - n\Delta t)$, and previous responses, $y(t - \Delta t)$ to $y(t - m\Delta t)$, together with the current white noise, $e(t)$.

Using the input and output signals to estimate the coefficients a_i and b_i is a problem of linear regression analysis. Therefore, when the signal-to-noise ratios in the seismic records are high enough, the ARX model will be the first choice among the other models because of its fast computing capability. Although increasing the orders, m and n, could reduce the identification error, estimating a_i and b_i will then become unstable when the orders are too large. Fortunately, the signal-to-noise ratios in our seismic records are always greater than 50, and it is shown that the ARX model is sufficient to describe the input–output relation when $m = n = 40$ [7].

In the frequency domain, the corresponding frequency transfer function is obtained by

$$H(\omega) = \frac{b_1 e^{-i\omega} + b_2 e^{-2i\omega} + \cdots + b_n e^{-ni\omega}}{1 + a_1 e^{-i\omega} + a_2 e^{-2i\omega} + \cdots + a_m e^{-mi\omega}} \tag{2}$$

where ω is the frequency. Using Equation (2) to estimate the frequency transfer function is one of the non-parametric identification problems for single-input–single-output systems. However, it is easy to determine the fundamental vibration period once $H(\omega)$ is identified.

IDENTIFICATION OF FUNDAMENTAL VIBRATION PERIOD

In each building, the recorded accelerations at the basement are considered to be the input signal, and all the accelerations at the upper measured floors are the output signals. Since there are

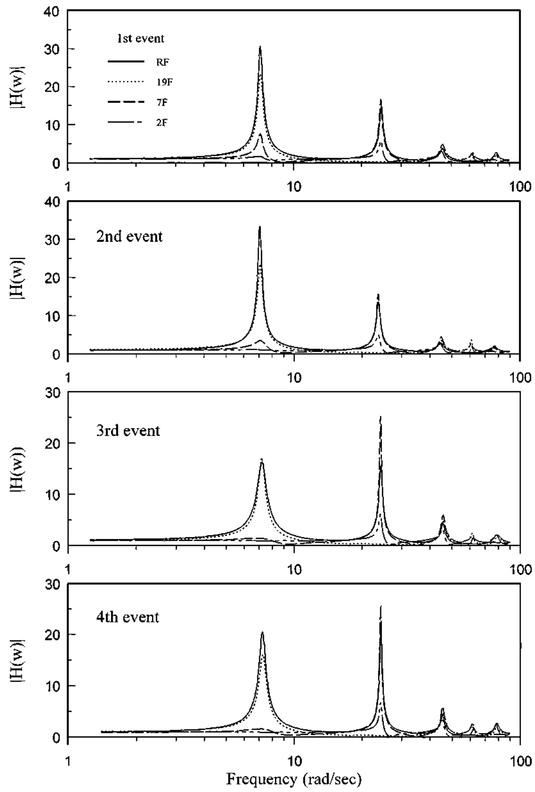

Figure 2. Absolute values of frequency transfer functions in longitudinal direction of building number 21.

several channels in the transverse direction, the recorded accelerograms are combined into one, by the interpolation (for two channels) or the regression (for three channels), at the rigid centre of each floor [7]. In the longitudinal direction, because there is usually only one channel located near the centre of each measured floor and the record is expected to be little affected by the floor rotation, the recorded accelerations are directly used as the input and output signals. Therefore, one frequency transfer function should be identified in each horizontal direction for each output floor.

Since the ARX model is used here to handle a simple single-input–single-output problem of identification, the rotational effect in the transverse records should be carefully considered, especially for the case that the fundamental frequency of the rotational mode is lower than that of the transverse mode. At first, the centre of the floor is assumed to be the rigid centre, so the transverse response at the centre and the rotational response of the floor are obtained through the process of interpolation or regression. The fundamental frequency of the rotational mode is then easily found when an ARX model is also used to identify the frequency transfer function of the rotational response. The rigid centre should be relocated if the transverse frequency transfer function at the centre is obviously affected by the rotational mode. The exact location of the rigid centre can be finally determined, by trial and error, when no peak in the transverse frequency transfer function occurs at the fundamental frequency of the rotational mode.

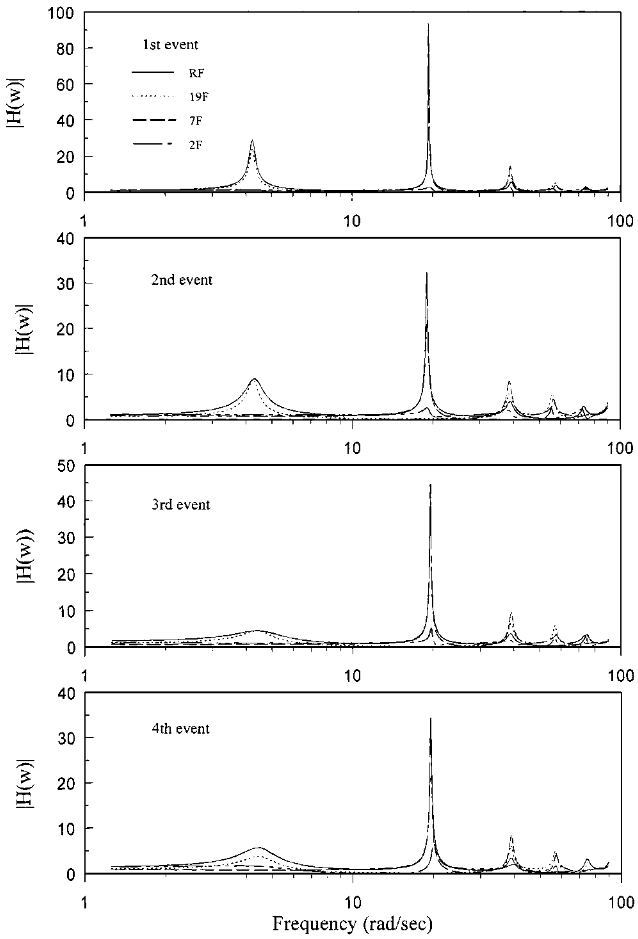

Figure 3. Absolute values of frequency transfer functions in transverse direction of building number 21.

For example, for the four events triggered by the building number 21, the identified frequency transfer functions in both two horizontal directions of all output floors are shown in Figures 2 and 3. In those figures, it is very clear to find the fundamental frequency, which is just the frequency corresponding to the first peak of $|H(\omega)|$. Not all $|H(\omega)|$ identified at the output floors of the other buildings have the same consistent fundamental frequencies as those in the building number 21. In general, the higher the output floor is, the larger the signal-to-noise ratio and $|H(\omega)|$ are. Hence, for each event the fundamental frequency is estimated by the following weighting average:

$$\omega_0 = \frac{\sum_{i=1}^{N} \omega_i |H_i(\omega_i)|}{\sum_{i=1}^{N} |H_i(\omega_i)|} \tag{3}$$

where N is the number of output floors, $H_i(\omega)$ is the identified frequency transfer function at the ith output floor, and ω_i is the frequency corresponding to the first peak of $|H_i(\omega)|$.

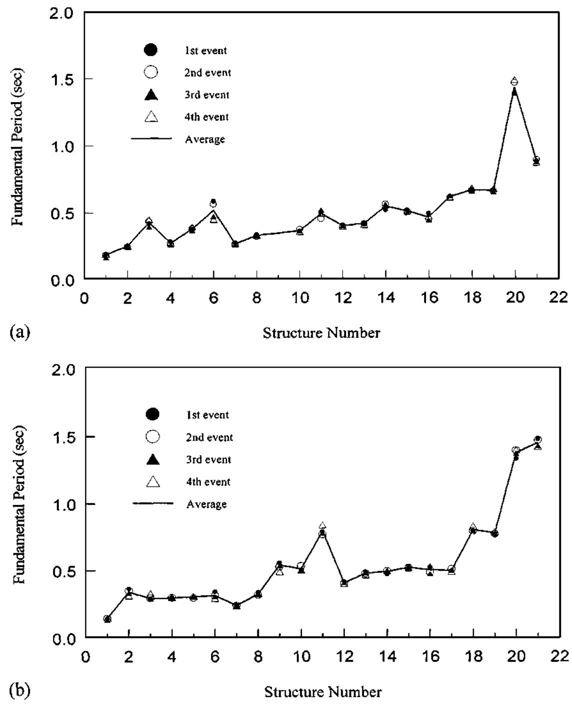

Figure 4. Fundamental vibration periods across events: (a) longitudinal direction, (b) transverse direction.

Four fundamental frequencies are obtained by Equation (3) for each building when the seismic records of the four events are used. To see the consistency of ω_0 across events, the fundamental vibration periods identified using the four events are shown in Figure 4 for each building. The averaged fundamental vibration periods across events are also listed in Table I. These averaged fundamental vibration periods of 21 RC MRF buildings constitute the database for determining the empirical formulas presented in the following paragraphs.

EMPIRICAL FORMULAS IN PREDICTING FUNDAMENTAL VIBRATION PERIOD

The simple and meaningful parameters in predicting the fundamental vibration period of a building are its height, length, width, and elapsed time after construction. Their contribution in predicting the fundamental vibration period is discussed as follows.

Height

In Table I all the heights of buildings are measured from the ground level to the roof. For building number 9, the floor plan is irregular and the only one longitudinal accelerometer on each floor is

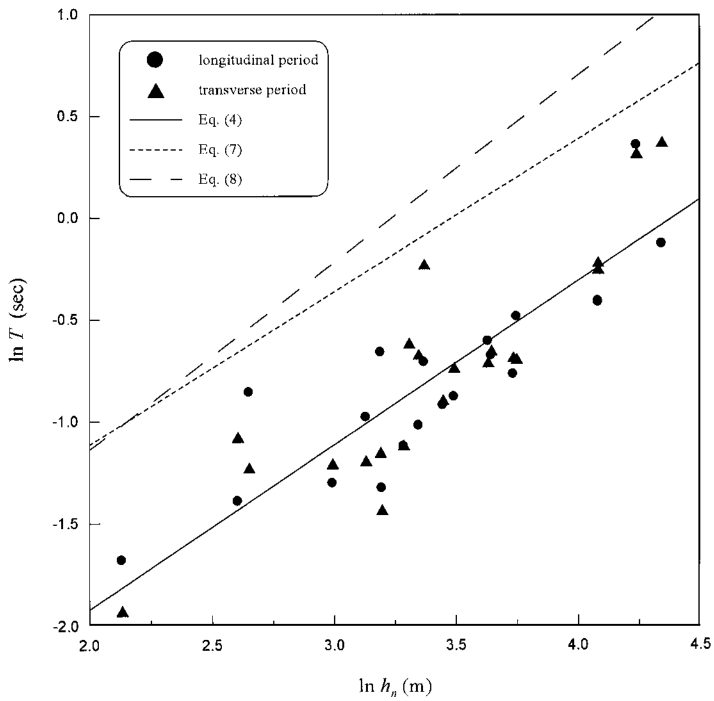

Figure 5. Identified periods and empirical formulas.

located at the edge of floor so that the identified frequency transfer functions are quite different among the four events. It is difficult to estimate the fundamental vibration period of this building in the longitudinal direction. Hence, there are total 41 identified periods in the two horizontal directions for those 21 buildings. Through a linear regression analysis to the 41 sample points of (h_n,ln T), where h_n is the height in m and T is the identified fundamental vibration period in s in either the longitudinal direction (T_l) or transverse direction (T_t) in Table I, the fundamental vibration period is predicted by

$$T = 0.0294\, h_n^{0.804} \tag{4}$$

where the unbiased estimate of the conditional standard deviation is $\sigma_{\ln T|h_n} = 0.253$. The log-normal distribution of T upon h_n is also verified at the 5 per cent significance level according to the Kolmogorov–Smirnov test. Therefore, Equation (4) predicts the median fundamental vibration period of a RC MRF building when its height is given. The predicted period with the exceedance probability other than 50 per cent can be easily obtained using Equation (4) and $\sigma_{\ln T|h_n}$. The identified periods and the regression line are shown in Figure 5.

If the regression analysis is performed to the identified periods in each horizontal direction separately, the results are

$$T_l = 0.0386 h_n^{0.717} \tag{5}$$

$$T_t = 0.0223 h_n^{0.891} \tag{6}$$

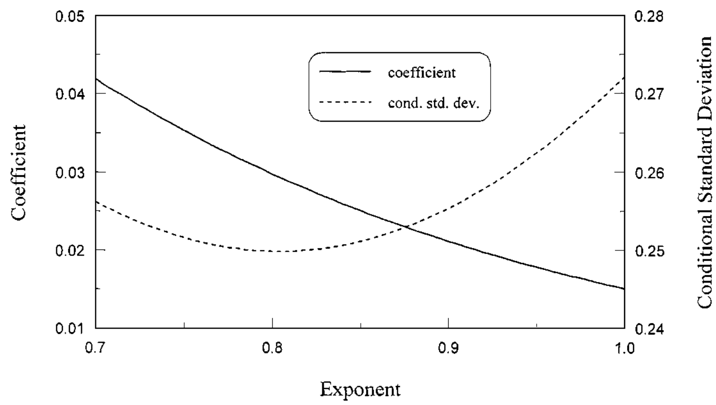

Figure 6. Sensitivity of the exponent in Equation (4).

where the unbiased estimate of the conditional standard deviation are $\sigma_{\ln T_1|h_n} = 0.239$ and $\sigma_{\ln T_t|h_n} = 0.267$. By comparing the conditional standard deviation, it is obvious that the prediction of the longitudinal fundamental vibration period is most reliable.

The UBC-97 provides the following formula to evaluate the fundamental vibration period of a RC MRF building [1]:

$$T = 0.0731\, h_n^{0.75} \tag{7}$$

The exponent in Equation (7) is a little lower than that in Equation (4), but the coefficient in Equation (7) is much higher than that in Equation (4). Hence, the code formula in the UBC-97 will overestimate the fundamental vibration periods of RC MRF buildings in Taiwan.

The formula presented in [6] which is obtained through a unconstrained regression analysis to the measured periods of RC MRF buildings in California, is

$$T_{50\%} = 0.0507\, h_n^{0.92} \tag{8}$$

It is noted that the period predicted in Equation (8) is always greater than that predicted in Equation (4). This comparison reflects a fact that the design and construction practices of RC MRF buildings are significantly different between Taiwan and California. Instead of using dry walls as practiced in California, it is a common practice in Taiwan that the interior walls are made of bricks and even RCs for quick construction. Hence, the buildings in Taiwan tend to be stiffer than those in California. The different empirical formulas in Equations (4), (7), and (8) are also compared in Figure 5.

It could be interesting to study the sensitivity of the exponent in Equation (4). The exponent is then constrained to a fixed value ranged from 0.70 to 1.00 and the coefficient and conditional standard deviation are estimated through the regression analysis. The results are shown in Figure 6.

Length and width

When the horizontal dimension of a building is additionally considered, the regressed formula becomes

$$T = 0.0344\, \frac{h_n^{0.803}}{D^{0.0429}} \tag{9}$$

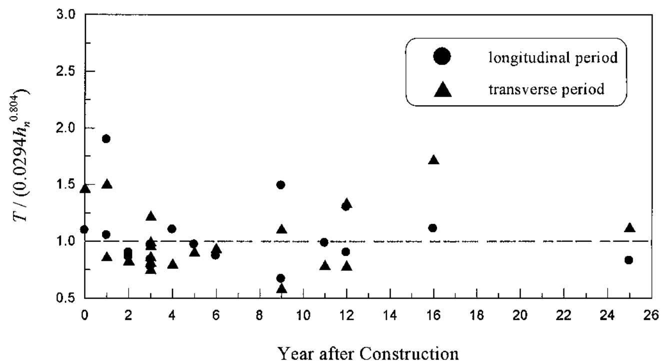

Figure 7. Ratios of identified period to predicted period.

where D is the length for $T = T_l$ or the width for $T = T_t$. In Equation (9) the estimated conditional standard deviation of ln T is 0.255. As compared with Equation (4), this value is a little higher than 0.253 due to the lower denominator in obtaining the unbiased estimate of 0.255. Obviously, the horizontal dimension of a RC MRF building plays a trivial role in predicting the fundamental vibration period because of the low exponent in D and no reduction of the conditional standard deviation, as compared with that in Equation (4).

Time after construction

Since all the measurements have been collected only for four years, it is difficult to observe the long-term variation of the identified periods for each building under normal use. However, the 21 RC MRF buildings were constructed at different times within a period of 25 years. The dispersion in Equation (4) offers the opportunity to decide if the time after completion of a building plays a deterministic role in predicting the fundamental vibration period. Mathematically, if the time after construction should be considered in Equation (4), then the empirical formula becomes

$$T = 0.0294\, h_n^{0.804} g(t) \tag{10}$$

Therefore, the ratios of the identified period to the one predicted in Equation (4) give the sample points of $g(t)$. Graphically, these ratios versus the times after construction are plotted in Figure 7. No monotonically varying trend is observed, so it is not necessary to consider the elapsed time after contruction in predicting the fundamental vibration period of a RC MRF building. In other words, the uncertainty in predicting the fundamental vibration period due to the time after construction has been implicitly included in $\sigma_{\ln T|h_n}$.

CONCLUSIONS

An empirical formula to predict the fundamental vibration period of RC MRF buildings in Taiwan is presented by using the recorded accelerograms of 21 such buildings. These buildings were under low-to-moderate shaking so that the linearly elastic responses were expected. The

identified frequency transfer function for each output floor is reliable by carefully relocating the seismic records to the rigid centre of each floor and choosing the orders of the ARX model. Then the fundamental vibration period of a building is estimated by the weighting average on the results of all output floors. The identified fundamental vibration period is acceptable in view of the almost repeatable results for different events triggered by the same building. Through the regression analysis, as expected, the height of a building plays the most important role, as compared to the horizontal dimension, on predicting the fundamental vibration period. The effect of the time after completion of buildings in predicting the fundamental vibration period could be covered by the standard deviation of the empirical formula. It is also found that the identified fundamental vibration periods of RC MRF buildings in Taiwan are obviously lower than those either measured in California or predicted in the code formula of the UBC-97. This is due to the fact that the design and construction practices of such building are significantly different between Taiwan and California.

ACKNOWLEDGEMENTS

This study was partially supported by the Central Weather Bureau, Ministry of Transportation and Communications, R.O.C. under Grant CWB 86-2E-11 and the National Science Council, R.O.C. under Grant NSC 86-2621-P-006-023. The authors are grateful to the Central Weather Bureau for providing the recorded accelerograms.

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